

- NURLAN KOGABAEV, *Computable dimension and projective planes.*

Sobolev Institute of Mathematics and Novosibirsk State University, Acad. Koptyug avenue 4, Novosibirsk 630090, Russia.

*E-mail:* kogabaev@math.nsc.ru.

In the present paper we investigate the question of possible computable dimensions of countable structures in the following familiar classes of projective planes: free projective planes, freely generated projective planes, pappian projective planes, and desarguesian projective planes.

For free projective planes the following characterization of computable categoricity have been found.

**THEOREM 1.** *Every free projective plane has computable dimension either 1 or  $\omega$ . Furthermore, such a plane is computably categorical if and only if it has finite rank.*

It turns out that the results of Theorem 1 can not be extended to the case of freely generated projective planes.

In [1] it was shown that the class of symmetric irreflexive graphs is *HKSS-complete* in the following computable-model-theoretic sense: for every countable structure  $\mathcal{A}$ , there exists a countable symmetric irreflexive graph  $\mathcal{G}$  which has the same degree spectrum as  $\mathcal{A}$ , the same  $\mathbf{d}$ -computable dimension as  $\mathcal{A}$  (for each degree  $\mathbf{d}$ ), the same computable dimension as  $\mathcal{A}$  under expansion by a constant, and which realizes every degree spectrum  $\text{DgSp}_{\mathcal{A}}(R)$  (for every relation  $R$  on  $\mathcal{A}$ ) as the degree spectrum of some relation on  $\mathcal{G}$ .

We construct an effective coding of symmetric irreflexive graphs into freely generated projective planes preserving most computable-model-theoretic properties and obtain the following result.

**THEOREM 2.** *The class of freely generated projective planes is HKSS-complete. In particular, for every  $n \in \omega \cup \{\omega\}$  there exists a freely generated projective plane of infinite rank with computable dimension  $n$ .*

In [2] it was proved that the class of fields is HKSS-complete. We use some natural coding of fields into pappian projective planes to obtain the following theorem.

**THEOREM 3.** *The class of pappian (desarguesian) projective planes is HKSS-complete. In particular, for every  $n \in \omega \cup \{\omega\}$  there exists a pappian (desarguesian) projective plane with computable dimension  $n$ .*

We also calculate the complexity of the computable categoricity problem for familiar classes of projective planes.

**THEOREM 4.** *The computable categoricity problem for the class of free projective planes is  $m$ -complete  $\Sigma_3^0$ . For the classes of freely generated, pappian, desarguesian and arbitrary projective planes the computable categoricity problem is  $m$ -complete  $\Pi_1^1$ .*

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[1] D.R.HIRSCHFELDT, B.KHOUSSAINOV, R.A.SHORE, A.M.SLINKO, *Degree spectra and computable dimensions in algebraic structures*, **Annals of Pure and Applied Logic**, vol. 115 (2002), no. 1-3, pp. 71–113.

[2] R.MILLER, B.POONEN, H.SCHOUTENS, A.SHLAPENTOKH, *A computable functor from graphs to fields*, **The Journal of Symbolic Logic**, vol. 83 (2018), no. 1, pp. 326–348.