▸ B.SH. KULPESHOV, *The countable spectrum of weakly o-minimal theories of finite convexity rank.*
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Here we discuss the Vaught's problem for weakly o-minimal theories of finite convexity rank. Convexity rank has been introduced in [1]. In particular, a theory has convexity rank 1 if there is no parametrically definable equivalence relation with an infinite number of infinite convex classes. Obviously, any o-minimal theory has convexity rank 1.

As it is known, in [2] the Vaught's conjecture for o-minimal theories was solved. Recently in [3] the Vaught's conjecture for quite o-minimal theories was solved. From the above works it follows that these theories have the same spectrum, namely such a theory has either continuum of countable models, or exactly $6^a 3^b$ countable models for non-negative integers a and b.

In [4] B.S. Baizhanov and A. Alibek have constructed for every ordinal κ with $4 \leq \kappa \leq \omega$ examples of weakly o-minimal theories having exactly κ countable models. All these examples have convexity rank 1. Recently in [5] the Vaught's conjecture for weakly o-minimal theories of convexity rank 1 was solved. Their countable spectrum differs from the countable spectrum of o-minimal theories. The following theorem describes the countable spectrum of weakly o-minimal theories of finite convexity rank (which coincides with the countable spectrum of weakly o-minimal theories of convexity rank 1):

THEOREM 1. *Let T be a weakly o-minimal theory of finite convexity rank in a countable language. Then exactly one of the following possibilities holds:*
(1) *T is $\aleph_0$-categorical*
(2) *T has k countable models, where $3 \leq k < \omega$*
(3) *T has ω countable models*
(4) *T has 2^ω countable models.*

[1] B.SH. KULPESHOV, *Weakly o-minimal structures and some of their properties*, ***The Journal of Symbolic Logic***, vol. 63, issue 4 (1998), pp. 1511–1528.

[2] L.L. MAYER, *Vaught's conjecture for o-minimal theories*, ***The Journal of Symbolic Logic***, vol. 53, issue 1 (1988), pp. 146–159.

[3] B.SH. KULPESHOV, S.V. SUDOPLATOV, *Vaught's conjecture for quite o-minimal theories*, ***Annals of Pure and Applied Logic***, vol. 168, issue 1 (2017), pp. 129–149.

[4] A. ALIBEK, B.S. BAIZHANOV, *Examples of countable models of a weakly o-minimal theory*, ***International Journal of Mathematics and Physics***, vol. 3, No. 2 (2012), pp. 1–8.

[5] A. ALIBEK, B.S. BAIZHANOV, B.SH. KULPESHOV, T.S. ZAMBARNAYA, *Vaught's conjecture for weakly o-minimal theories of convexity rank 1*, ***Annals of Pure and Applied Logic***, vol. 169, issue 11 (2018), pp. 1190–1209.