

# A REPORT ON NSOP<sub>1</sub> THEORIES

BYUNGHAN KIM

ABSTRACT. This is a joint work with J. Dobrowolski and N. Ramsey. SOP<sub>1</sub> (i.e., the strong order proper 1) is introduced by S. Shelah, and it implies the tree property. Hence any simple theory is NSOP<sub>1</sub> (i.e., not having SOP<sub>1</sub>). The random parametrized equivalence relations, an infinite dimensional vector space with a bilinear map, and an unbounded PAC field are typical examples having non-simple NSOP<sub>1</sub> theories. Recently I. Kaplan and N. Ramsey showed that in any NSOP<sub>1</sub> theory, **over models**, Kim-independence satisfies all the basic axioms that nonforking satisfies in simple theories (such as symmetry, transitivity lifting, local character, extension, and type-amalgamation), except base monotonicity. (In simple theories, Kim-independence and nonforking independence coincide.)

Now we show that the same hold **over any set**. Namely in any NSOP<sub>1</sub> theory, over any set, Kim-independence satisfies all the mentioned basic axioms (except base monotonicity) including type-amalgamation of Lascar types. There need several steps to prove this and recently we finally show a step that any NSOP<sub>1</sub> theory holds nonforking existence, so now the result is unconditional. I will talk about other related topics/results as well.