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FURTHER RESULTS ON HEILBERT'S TENTH PROBLEM

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Abstract

Hilbert's Tenth Problem (HTP) asks for an effective algorithm to test whether an arbitrary polynomial equation

$$P(x_1, \cdots, x_n) = 0$$

(with integer coefficients) has solutions over the ring $\mathbb{Z}$ of the integers. This was finally solved by Matiyasevich in 1970 negatively.

In this talk we introduce the speaker's further results on HTP. We present a sketch of the proof of the speaker's main result that there is no effective algorithm to determine whether an arbitrary polynomial equation $P(x_1, \ldots, x_{11}) = 0$ (with integer coefficients) in 11 unknowns has integral solutions or not. We will also mention some other results of the speaker, for example, there is no algorithm to test for any $P(z_1, \ldots, z_{17}) \in \mathbb{Z}[z_1, \ldots, z_{17}]$ whether $P(z_1^2, \ldots, z_{17}^2) = 0$ has integral solutions, and also there is a polynomial $Q(z_1, \ldots, z_{20}) \in \mathbb{Z}[z_1, \ldots, z_{20}]$ such that

$$\{Q(z_1^2, \ldots, z_{20}^2) : \ z_1, \ldots, z_{20} \in \mathbb{Z}\} \cap \{0, 1, 2, \ldots\}$$

coincides with the set of all primes.