Universal trees are very basic combinatorial objects: an ordered tree is \((n, h)\)-universal if every ordered tree of height at most \(h\) and with at most \(n\) leaves can be embedded into it. We give nearly-matching upper and lower bounds on the size of the smallest \((n, h)\)-universal trees: if \(h\) is asymptotically logarithmic in \(n\) then it is polynomial in \(n\), and if \(h\) is asymptotically super-logarithmic in \(n\) then it is quasi-polynomial.

We then discuss several applications of universal trees: - to design algorithms for solving parity games and mean-payoff parity games, which are quasi-polynomial and pseudo-quasi-polynomial, respectively; - to show that separating automata—due to Bojanczyk and Czerwinski—unify all (but one) currently known quasi-polynomial algorithms for solving parity games: due to Calude, Jain, Khoussainov, Li, and Stephan, 2017; Jurdzinski and Lazic, 2017; Fearnley, Jain, Schewe, Stephan, and Wojtczak, 2017; and Lehtinen, 2018; - to prove a quasi-polynomial lower bound on the size of all strongly separating automata, hence pinpointing a quasi-polynomial barrier that most existing techniques for solving parity games efficiently are vulnerable to; - to improve and streamline translations from alternating parity automata on words to alternating weak automata.

The talk is based on joint work with various subsets of: Wojciech Czerwinski, Laure Daviaud, Nathanael Fijalkow, Ranko Lazic, Karoliina Lehtinen, and Pawel Parys.